Chapter 6 Notes Name Key (2014)

6.1 Slope Fields

Example: Find the <u>general solution</u> to the differential equation $dy/dx = 9x^2 - 4x + 5$.

find equation: y= ~

1) separate variables $dy = (9x^2 - 4x + 5)dx$

2) Integrate: $\int dy = \int (9x^2 - 4x + 5)dx$

$\boxed{y = 3x^3 - 2x^2 + 5x + c}$ ← check w/ derivative

Example: <u>Solve the initial value problem</u> when $f(-1) = 0$ for $dy/dx = 9x^2 - 4x + 5$.

find equation: y= ~
find c

1) See above
2)
3) $0 = 3(-1)^3 - 2(-1)^2 + 5(-1) + c$

$10 = c$

$\boxed{y = 3x^3 - 2x^2 + 5x + 10}$

Example: <u>Solve the initial value problem</u> when $y(\pi) = 1$ for $dy/dx = sinx + cosx$.

Separate & integrate

$\int dy = \int (\sin x + \cos x)dx$

$y = -\cos x + \sin x + c$

$1 = -\cos \pi + \sin \pi + c$

$1 = -(-1) + 0 + c$

$0 = c$

$\boxed{y = -\cos x + \sin x}$

Slope Field for a first order differential equation dy/dx = f(x,y):
Plotting short line segments w/ slopes dy/dx at lattice points – shows "family of solutions"

why needed? When sol. to diff. eq. is impossible

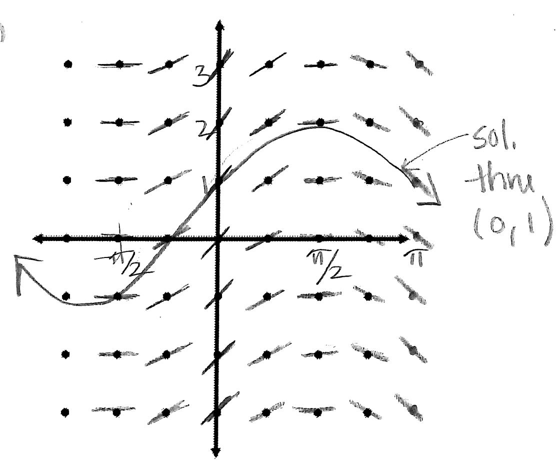
Example: a) Create a <u>slope field</u> for the differential equation $dy/dx = cosx$.

slope equation

X / Y	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
-2	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
-1							
0							
1							
2							

find slope when $x = \pi$ and $y = -2$

$\frac{dy}{dx} = \cos\pi = -1$



b) Find the <u>general solution</u> to the exact differential equation.

$\int dy = \int \cos x\,dx$

$\boxed{y = \sin x + c}$

slope field shows this "family of solutions" slope equation

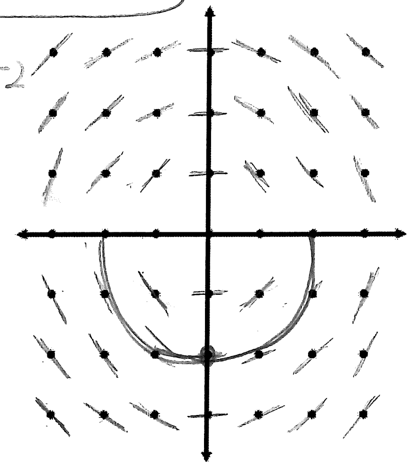
Example: a) Construct a slope field for the differential equation $\boxed{dy/dx = -x/y.}$

y \ x	-3	-2	-1	0	1	2	3
-3	-1	$-2/3$	$-1/3$	0	$1/3$	$2/3$	1
-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$
-1	-3	-2	-1	0	1	2	3
0	—	—	—	—	—	—	—
1	3	2	1	0	-1	-2	-3
2	$3/2$	1	$1/2$	0	$-1/2$	-1	$-3/2$
3	1	$2/3$	$1/3$	0	$-1/3$	$-2/3$	-1

Slope when $x=3, y=-2$

$\frac{dy}{dx} = \frac{-3}{-2} = 3/2$

graph nothing - could be corner, cusp, vert. tangent



b) Solve the initial value problem using $(0, -2)$. Sketch the graph of the particular solution.

$\int y\,dy = \int -x\,dx$

* find c right away →

$\frac{y^2}{2} = \frac{-x^2}{2} + C \quad \rightarrow \quad \frac{y^2}{2} = \frac{-x^2}{2} + 2$

$\frac{(-2)^2}{2} = \frac{0^2}{2} + C \qquad y^2 = -x^2 + 4$

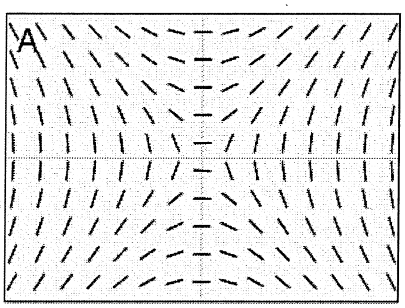
$2 = C \qquad y = \pm\sqrt{-x^2+4}$

not both, find which using $(0,-2)$

$-2 = \ominus\sqrt{-0^2+4}$

$\boxed{y = -\sqrt{-x^2+4}}$

Example: Match the differential equations to the slope field.



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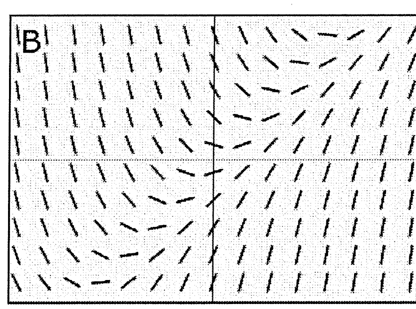

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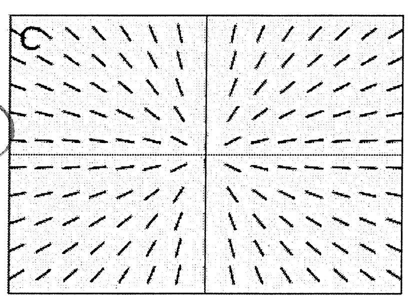

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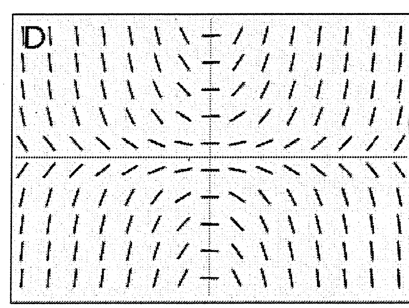

③

can plug in any values

	1. $\frac{dy}{dx} = x - y$	**2.** $\frac{dy}{dx} = xy$	**3.** $\frac{dy}{dx} = \frac{x}{y}$	**4.** $\frac{dy}{dx} = \frac{y}{x}$
$x=0$ (y axis)	$m = -y$ B	$m = 0$ A or D	$m = 0$ A or D	$m = $ und C
$y=0$ (x axis)	$m = x$	$m = 0$ A	$m = $ und D	$m = 0$

6.2: Antidifferentiation by Substitution

Indefinite Integrals: no limits of integration, need "+ c"

Properties:

1. $\int kf(x)dx =$ $K \cdot \int f(x)dx$
2. $\int (f(x) \pm g(x))dx =$ $\int f(x)dx \pm \int g(x)dx$

Examples:

$\int (x^3 + x)dx = \frac{x^4}{4} + \frac{x^2}{2} + c$

$\int \left(\frac{\cos x}{3}\right) dx = \frac{1}{3}\int \cos x dx = \boxed{\frac{1}{3}\sin x + c}$

$\int (5/x)\, dx = 5\int \frac{1}{x} dx = \boxed{5\ln x + c}$

$\int \left(e^x - \frac{1}{x} + \cos x\right) dx = \boxed{e^x - \ln|x| + \sin x + c}$

Integration by Substitution: → "u-substitution"

- try: u= inside function
- try: u= denominator

Examples:

$\int (2x - 3)^3\, dx = \int u^3 \cdot \frac{1}{2} du$

$u = 2x-3$

$du = 2dx \rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\int u^3 du = \frac{1}{2} \cdot \frac{u^4}{4} + c = \boxed{\frac{1}{8}(2x-3)^4 + c}$

$\int \sqrt{5x - 3}\, dx = \int \sqrt{u} \cdot \frac{1}{5} du$

$u = 5x-3$

$du = 5dx$

$\frac{1}{5}du = dx$

$= \frac{1}{5}\int u^{1/2} du = \frac{1}{5} \cdot \frac{u^{3/2}}{3/2} + c$

$= \frac{1}{5} \cdot \frac{2}{3}(5x-3)^{3/2} + c = \boxed{\frac{2}{15}(5x-3)^{3/2} + c}$

$\int x(x^2 + 1)^2\, dx = \int (x^2+1)^2 \cdot x dx$

$u = x^2+1$

$du = 2x dx$

$\frac{1}{2}du = x dx$

$= \int u^2 \cdot \frac{1}{2} du = \frac{1}{2}\int u^2 du$

$= \frac{1}{2} \cdot \frac{u^3}{3} + c = \boxed{\frac{1}{6}(x^2+1)^3 + c}$

"u-sub": good for composite functions & products

Substitution Method: 3 "Easy" Steps

1 determine: $u=$ ______
and $du=$ ______ } sub in for x's

2 Integrate

3 Sub x's back in

* "du" always multiplied (never in denominator, exponent, etc.)

More Examples:

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{1}{u}\cdot du = -\ln|u| + c$$

$$\boxed{= -\ln|\cos x| + c}$$

$u = \cos x$

$du = -\sin x \cdot dx$

$$\int (x^2+1)^4 2x\,dx = \int u^4 \cdot du = \frac{u^5}{5} + c$$

$$= \boxed{\frac{(x^2+1)^5}{5} + c}$$

$u = x^2+1$

$du = 2x\,dx$

$$\int \frac{x}{(x^2+9)}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + c$$

$$= \frac{1}{2}\ln|x^2+9| + c = \boxed{\ln\sqrt{x^2+9} + c}$$

$u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\cdot dx$

$$\int \frac{1}{x^2+9}dx$$

looks like: $\frac{d}{dx}\tan^{-1}u = \frac{1}{u^2+1}\,du/dx$

must be a 1, so factor out 9

$u = x^2+9$
$du = 2x\,dx$ } doesn't work ☹

$$= \int \frac{1}{9\left(\frac{x^2}{9}+1\right)}dx = \frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}dx$$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3du = dx$

$$= \frac{1}{9}\cdot 3\int \frac{1}{u^2+1}du = \frac{3}{9}\tan^{-1}u + c$$

$$= \boxed{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c}$$

have limits of integration

u-**Substitution with "Definite Integrals":**

To evaluate:

1) switch limits of integration & never go back to x's

OR

2) don't change limits, sub x's back in after integrating

Examples:

$$\int_{x=-1}^{x=3} (x^2+1)^{-1}x\,dx = \frac{1}{2}\int_{u=2}^{u=10} u^{-1}\,du = \frac{1}{2}\int_2^{10} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_2^{10}$$

$$= \frac{1}{2}(\ln 10 - \ln 2) = \frac{1}{2}\ln\left(\frac{10}{2}\right)$$

$$= \boxed{\ln\sqrt{5}}$$

$u = x^2+1$

$du = 2x \cdot dx$

$\frac{1}{2}du = dx$

limits:

$u = (-1)^2+1 = 2$

$u = (3)^2+1 = 10$

method #1

$$\int_{x=0}^{x=\pi/2} \sin^2 x \cos x\,dx = \int_{x=0}^{x=\pi/2} u^2 \cdot du = \frac{u^3}{3}\Big|_{x=0}^{x=\pi/2} = \frac{(\sin x)^3}{3}\Big|_0^{\pi/2}$$

$$= \frac{(\sin \frac{\pi}{2})^3}{3} - \frac{(\sin 0)^3}{3}$$

$$= \frac{1^3}{3} - \frac{0^3}{3} = \boxed{\frac{1}{3}}$$

$u = \sin x$

$du = \cos x \cdot dx$

method #2

$$\int_0^{\pi/4} \sin^2 x\,dx = \int_0^{\pi/4} \frac{1}{2}(1-\cos 2x)\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos u \cdot du$$

$$= \frac{1}{2}x - \frac{1}{4}\sin u\Big|_{x=0}^{x=\pi/4}$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x\Big|_0^{\pi/4}$$

$$= \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\sin\left(2\cdot\frac{\pi}{4}\right) - \left(\frac{1}{2}\cdot 0 - \frac{1}{4}\sin 0\right)$$

$$= \frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}$$

$$= \boxed{\frac{\pi}{8} - \frac{1}{4}}$$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

* u-sub doesn't work ... now what ??

Power Reduction Identities

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

need these for $\int \sin^2 x\,dx$ or $\int \cos^2 x\,dx$

6.3: Integration by Parts

→ try this when u-sub doesn't work

→ good for integrating products

- product rule:
  $\frac{d}{dx}(u \cdot v) = u \cdot dv + v du$
- integrate both sides:
  $\int \frac{d}{dx}(u \cdot v) = \int u dv + \int v du$
- rearrange:

$$\boxed{\int u dv = uv - \int v du}$$

Integration By-Parts Formula

choose what u= in this order

- L Logs ($\ln x$)
- I Inverse trig ($\tan^{-1} x$)
- P Polynomials ($3x^2$)
- E Exponentials ($2^x, e^x$)
- T Trig ($\sin x$)

Examples:

poly, log

$$\int x^2 \ln x dx = \int \overbrace{\ln x}^{u} \cdot \overbrace{x^2 dx}^{dv} = \underbrace{\ln|x| \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx}_{\text{simplify!}}$$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$

$$= \ln|x| \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx$$

$$= \frac{1}{3} x^3 \ln|x| - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

$$= \boxed{\frac{1}{3} x^3 \ln|x| - \frac{1}{9} x^3 + C}$$

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poly, trig

$$\int \overbrace{3x}^{u} \overbrace{\sin 4x dx}^{dv} = \underbrace{3x \cdot -\frac{1}{4} \cos 4x - \int -\frac{1}{4} \cos 4x \cdot 3 dx}_{\text{simplify!}}$$

$u = 3x \qquad dv = \sin 4x \cdot dx$

$du = 3 dx \qquad v = -\frac{1}{4} \cos 4x$

need u-sub for $\int \sin 4x dx$

$u = 4x \qquad = \frac{1}{4} \int \sin u du$

$du = 4dx$

$\frac{1}{4} du = dx \qquad = -\frac{1}{4} \cos u$

$= -\frac{1}{4} \cos 4x$

$$= \frac{-3x}{4} \cos 4x + \frac{3}{4} \int \cos 4x dx$$

$$= -\frac{3}{4} x \cos 4x + \frac{3}{4} \cdot \frac{1}{4} \sin 4x + C$$

$$= \boxed{-\frac{3}{4} x \cos 4x + \frac{3}{16} \sin 4x + C}$$

poly trig trig poly simplify!

$$\int x^2 \cos x\,dx = x^2\sin x - \int \sin x \cdot 2x\,dx = x^2\sin x - \left(2x(-\cos x) - \int -\cos x \cdot 2\,dx\right)$$

$$= x^2\sin x + 2x\cos x - 2\int \cos x\,dx$$

$$= \boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$$

I: $u = x^2$, $dv = \cos x\,dx$; $du = 2x\,dx$, $v = \sin x$

II: $u = 2x$, $dv = \sin x\,dx$; $du = 2\,dx$, $v = -\cos x$

Tabular Integration: Shortcut for By-Parts Method

Must have: $\int (\text{Polynomial})\left(\begin{smallmatrix}\text{something}\\ \text{easy to integrate}\\ \text{repeatedly}\end{smallmatrix}\right)dx$ — like $\sin x$, $\cos x$, e^x, ...

$\int x^2 \cos x\,dx$

	D	I
+	x^2	$\cos x$
−	$2x$	$\sin x$
+	2	$-\cos x$
−	0	$-\sin x$

$$= \boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$$

don't forget!

$$\int e^{2x}x^3\,dx = x^3\cdot\tfrac{1}{2}e^{2x} - 3x^2\cdot\tfrac{1}{4}e^{2x} + 6x\cdot\tfrac{1}{8}e^{2x} - 6\cdot\tfrac{1}{16}e^{2x} + C$$

$$= \boxed{e^{2x}\left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8}\right) + C}$$

	D	I
+	x^3	e^{2x}
−	$3x^2$	$\frac{1}{2}e^{2x}$
+	$6x$	$\frac{1}{4}e^{2x}$
−	6	$\frac{1}{8}e^{2x}$
+	0	$\frac{1}{16}e^{2x}$

exponential trig

Tabular does not work!

$\int e^x \cos x\, dx$

I $\begin{cases} u = e^x & dv = \cos x\,dx \\ du = e^x dx & v = \sin x \end{cases}$ $= e^x \sin x - \int \sin x \cdot e^x dx$

integral not "better"... by parts again

simplify!

II $\begin{cases} u = e^x & dv = \sin x\,dx \\ du = e^x dx & v = -\cos x \end{cases}$ $= e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x\,dx\right]$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$
$$+\int e^x \cos x\,dx \qquad\qquad +\int e^x \cos x\,dx$$

"endless integration"

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$
$$\int e^x \cos x\,dx = \boxed{\tfrac{1}{2}(e^x \sin x + e^x \cos x) + C}$$

don't forget!

$$\int_4^1 \ln x\,dx = \ln|x| \cdot x - \int x \cdot \frac{1}{x}\,dx = \ln|x| \cdot x - \int 1\,dx$$

$\begin{cases} u = \ln x & dv = 1 \cdot dx \\ du = \frac{1}{x}dx & v = x \end{cases}$

$$= x \ln|x| - x \Big|_4^1$$
$$= 1 \cdot \underbrace{\cancel{\ln 1}}_{0} - 1 - [4\ln 4 - 4]$$
$$= -1 - 4\ln 4 + 4$$
$$= 3 - 4\ln 4$$
$$= 3 - \ln 4^4 = \boxed{3 - \ln 256}$$

When Integrating, don't try "u-sub" or "by-parts" first! Look at function... can it be simplified/morphed?

$$\int \frac{x^3 - 7x + 4}{x^{1/2}}\,dx$$

$$= \int \left(\frac{x^3}{x^{1/2}} - \frac{7x}{x^{1/2}} + \frac{4}{x^{1/2}}\right)dx = \int \left(x^{5/2} - 7x^{1/2} + 4x^{-1/2}\right)dx$$
$$= \frac{2}{7}x^{7/2} - 7 \cdot \frac{2}{3}x^{3/2} + 4 \cdot 2x^{1/2} + C$$
$$= \boxed{\frac{2}{7}x^{7/2} - \frac{14}{3}x^{3/2} + 8x^{1/2} + C}$$

6.4: Exponential Growth and Decay

Solve: $\frac{dy}{dt} = k \cdot y$

$\int \frac{1}{y} dy = \int k \cdot dt$

$e^{\ln|y|} = e^{kt+c}$

$|y| = e^{kt} \cdot e^{c}$

constant, let's call it "P"

$|y| = Pe^{kt}$

$y = Pe^{rt}$

constant could be neg. or pos.

This is where "Pe^{rt}" comes from... it's the solution to this differential eqn. $dy/dt = k \cdot y$

Law of Exponential Change:

A quantity (population, radioactive element, $) that increases or decreases at a rate proportional to the amount present can be represented by a differential equation:

$$\frac{dy}{dt} = ky \xrightarrow{\text{solution}} y = y_0 e^{kt}$$

"rate of increase proportional to amt present"

1. Compounding Continuously: $A = Pe^{rt}$

A = final amt
P = start amt (principal)
r = rate as decimal
t = time

2. Newton's Law of Cooling: $T - T_s = (T_0 - T_s)e^{-kt}$

T = final temp
T_0 = start temp
T_s = surrounding temp
k = constant (rate depends on substance cooling)
t = time

3. Radioactive Decay: $y = y_0 e^{-kt}$

y = final amt
y_0 = start amt
k = constant of decay (rate)
t = time

Example: Find the solution of the differential equation $dy/dt = ky$ that satisfies $y(0) = 50$ and $y(5) = 100$.

solution: $y = Pe^{kt}$ or $y = y_0 e^{kt}$ (find y_0 and k)

$100 = 50e^{k(5)}$

$2 = e^{5k}$

$\ln 2 = \ln e^{5k}$

$\ln 2 = 5k$

$k = \frac{\ln 2}{5}$

$\boxed{y = 50e^{\frac{\ln 2}{5}t}}$

Example:

a) In general, how long does it take to double your money if it is compounded continuously?

$y = Pe^{rt}$

$2 = 1e^{rt}$

$2 = e^{rt}$

$\ln 2 = \ln e^{rt}$

$\ln 2 = rt$

$\boxed{t = \frac{\ln 2}{r}}$

b) What is the interest rate if $2000, compounded continuously, doubles in 15 years?

$y = Pe^{rt}$

$4000 = 2000e^{r(15)}$

$2 = e^{15r}$

$\ln 2 = \ln e^{15r}$

$\ln 2 = 15r$

$\boxed{r = \frac{\ln 2}{15}}$

c) How much will the $2000 be worth after 30 years?

$\$2000 \xrightarrow[\text{dbls in 15yrs}]{} \$4000 \xrightarrow[\text{dbls in 15yrs}]{} \$8{,}000$

Example: Suppose that the cholera bacteria grows unchecked according to the Law of Exponential Growth. A colony starts with 1 bacterium and doubles every 30 minutes. How many bacteria will the colony contain at the end of 24 hours?

0.5 hours

$y = y_0 e^{kt}$

$2 = 1e^{k(.5)}$

$\ln 2 = \ln e^{.5k}$

$\ln 2 = .5k$

$k = \frac{\ln 2}{.5} = 2\ln 2 = \ln 4$

$y = 1 \cdot e^{\ln 4 (24)}$

$y = \boxed{2.815 \times 10^{14} \text{ bacteria}}$

Example: The number of radioactive atoms remaining after t days ina sample of polonium-210 that starts with y_0 radioactive atoms is $y = y_0 e^{-.005t}$.

•What is the half life?

amt. of time takes for $\frac{1}{2}$ to decay

$\frac{1}{2} = 1e^{-.005t}$

$\ln \frac{1}{2} = \ln e^{-.005t}$

$\ln \frac{1}{2} = -.005t$

$\boxed{t = \frac{\ln 1/2}{-.005}}$ ≈ 138.629 days

•When will there be 5% of the original amount remaining?

$y = y_0 e^{kt}$

$.05 = 1e^{-.005t}$

$\ln .05 = \ln e^{-.005t}$

$\ln .05 = -.005t$

$\boxed{t = \frac{\ln .05}{-.005}}$ ≈ 599.146 days

T_0 T t

Example: Suppose a cup of soup cooled from 90°C to 60°C in 10 minutes in a room whose temperature was 20°C. How much longer would it take to cool to 35°C?

T_s

final amt (T)
2nd part of prob

$$T - T_s = (T_0 - T_s)e^{-kt}$$

$$60 - 20 = (90-20)e^{-k(10)}$$

$$40 = 70e^{-10k}$$

$$\frac{4}{7} = e^{-10k}$$

$$\ln\frac{4}{7} = -10k$$

$$k = \frac{\ln\frac{4}{7}}{-10}$$

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now: $T = 35^\circ$
$T_0 = 60^\circ$

$$35 - 20 = (60-20)e^{-\left(\frac{\ln\frac{4}{7}}{-10}\right)t}$$

$$15 = 40e^{\frac{\ln\frac{4}{7}}{10}t}$$

$$\frac{15}{40} = e^{\frac{\ln\frac{4}{7}}{10}t}$$

$$\ln\left(\frac{15}{40}\right) = \frac{\ln\left(\frac{4}{7}\right)}{10}t$$

$$\frac{10\cdot\ln\left(\frac{15}{40}\right)}{\ln\left(\frac{4}{7}\right)} = t$$

$$t \approx \boxed{17.527 \text{ min}}$$

Examples: Solve the initial value problems. → separate & integrate

$dy/dx = 2x/y$ at $(1, -3)$.

$y\,dy = 2x\,dx$

$\int y\,dy = \int 2x\,dx$

$\frac{y^2}{2} = x^2 + C \longrightarrow \frac{y^2}{2} = x^2 + \frac{7}{2}$

$\frac{(-3)^2}{2} = 1^2 + C \qquad y^2 = 2x^2 + 7$

$\frac{9}{2} - 1 = C \qquad y = \pm\sqrt{2x^2+7}$

$C = 7/2$

not both... check w/ (1,−3)

$-3 \stackrel{?}{=} \pm\sqrt{2(1)^2+7}$

$-3 \stackrel{?}{=} \pm\sqrt{9}$

$\boxed{y = -\sqrt{2x^2+7}}$

$dy/dx = 100x - xy$, and $y = 1$ when $x = 2$.

$\frac{dy}{dx} = x(100-y)$

$\int \frac{1}{100-y}\,dy = \int x\,dx$

$u = 100-y$
$du = -dy$

$-\ln|100-y| = \frac{x^2}{2} + C \longrightarrow -\ln|100-y| = \frac{x^2}{2} - \ln 99 - 2$

$-\ln|100-1| = \frac{2^2}{2} + C \qquad e^{(\ln|100-y|)} = e^{(2+\ln 99 - \frac{x^2}{2})}$

$-\ln(99) - 2 = C$

$|100-y| = e^2 \cdot e^{\ln 99} \cdot e^{-x^2/2}$

$100 - y = \pm 99e^{2-\frac{x^2}{2}}$

check which

$100 - 1 \stackrel{?}{=} \pm 99e^{2-\frac{2^2}{2}}$

$99 = \pm 99(e^0)$

$100 - y = +99e^{2-\frac{x^2}{2}}$

$-y = 99e^{2-\frac{x^2}{2}} - 100$

$\boxed{y = 100 - 99e^{2-\frac{x^2}{2}}}$